Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|
| | |

**Pearson Edexcel International GCSE**

Centre Number | | | | | |

Candidate Number | | | | |

# Monday 17 June 2019

Afternoon (Time: 2 hours) | Paper Reference **4PM1/01**

# Further Pure Mathematics

## Paper 1

**Calculators may be used.**

Total Marks

## Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

## Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

*Turn over* ▶

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Pearson

# International GCSE in Further Pure Mathematics Formulae sheet

## Mensuration

**Surface area of sphere** = $4\pi r^2$

**Curved surface area of cone** = $\pi r \times$ slant height

**Volume of sphere** = $\frac{4}{3}\pi r^3$

## Series

### Arithmetic series

Sum to $n$ terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

### Geometric series

Sum to $n$ terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

### Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

## Calculus

### Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

## Trigonometry

### Cosine rule

In triangle $ABC$: $a^2 = b^2 + c^2 - 2bc\cos A$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$ $\quad$ $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$ $\quad$ $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \qquad \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

## Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

**Answer all ELEVEN questions.**

**Write your answers in the spaces provided.**

**You must write down all the stages in your working.**

**1** $f(x) = x^3 + 2x^2 - 5x - 6$

(a) Factorise $x^2 - x - 2$

**(1)**

(b) Hence, or otherwise, show that $(x^2 - x - 2)$ is a factor of $f(x)$.

**(3)**

**(Total for Question 1 is 4 marks)**

**2** Given that $\frac{4+2\sqrt{3}}{5-2\sqrt{3}}$ can be written in the form $\frac{a+b\sqrt{3}}{c}$ where $a$ and $b$ are integers and $c$ is prime, find the value of $a$, the value of $b$ and the value of $c$.

**Show your working clearly**.

**(3)**

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**Question 2 continued**

**(Total for Question 2 is 3 marks)**

**3** In triangle $ABC$, $AC = 7\,\text{cm}$, $BC = 10\,\text{cm}$ and angle $BAC = 65°$

(a) Find, to the nearest 0.1°, the size of angle $ABC$.

**(3)**

(b) Find, in $\text{cm}^2$ to 3 significant figures, the area of triangle $ABC$.

**(3)**

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**Question 3 continued**

**(Total for Question 3 is 6 marks)**

**4**

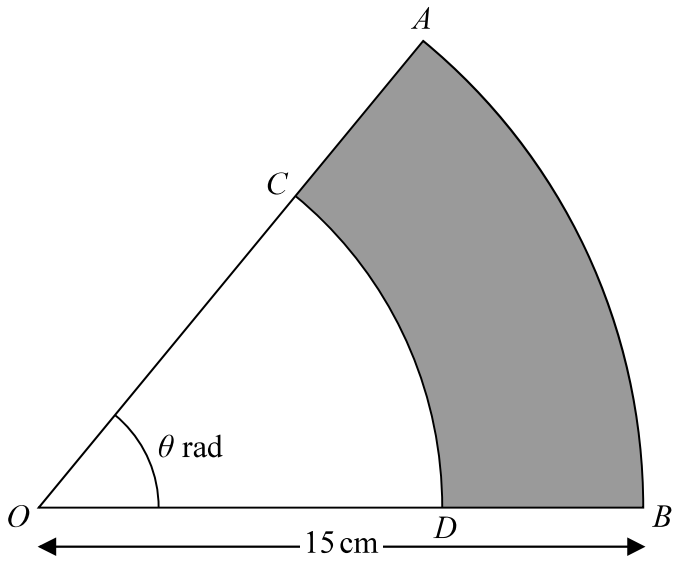


Diagram **NOT** accurately drawn

**Figure 1**

Figure 1 shows a sector $OAB$ of a circle where angle $AOB = \theta$ radians. The circle has centre $O$ and radius 15 cm. The point $C$ divides $OA$ in the ratio 2 : 1 and the point $D$ divides $OB$ in the ratio 2 : 1

The area of the region $ABDC$, shown shaded in Figure 1, is 100 cm$^2$

Find

(a) the value of $\theta$,

**(3)**

(b) the perimeter of the region $ABDC$.

**(3)**

**Question 4 continued**

**(Total for Question 4 is 6 marks)**

**5** $f(x) = 3x^2 - 9x + 5$

Given that $f(x)$ can be written in the form $a(x - b)^2 + c$, where $a$, $b$ and $c$ are constants, find

(a) the value of $a$, the value of $b$ and the value of $c$.

**(3)**

(b) Hence write down

(i) the minimum value of $f(x)$,

(ii) the value of $x$ at which this minimum occurs.

**(2)**

**Question 5 continued**

**(Total for Question 5 is 5 marks)**

**6**

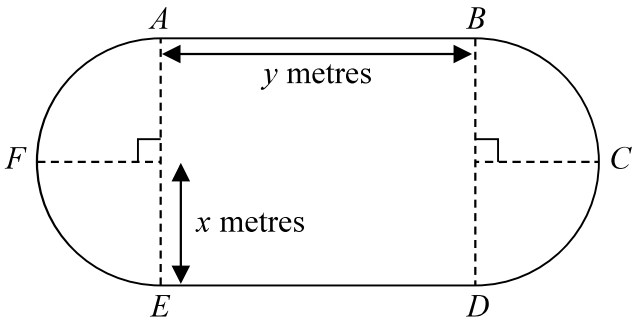


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**Figure 2**

Figure 2 shows a lawn $ABCDEF$, where $ABDE$ is a rectangle of length $y$ metres and width $2x$ metres. Each end of the lawn is a semicircle of radius $x$ metres. The lawn has perimeter 90 m and area $S$ m$^2$

(a) Show that $S$ can be written in the form

$$S = kx - \pi x^2$$

where $k$ is a constant.

State the value of $k$.

**(4)**

(b) Use calculus to find, to 4 significant figures, the value of $x$ for which $S$ is a maximum, justifying that this value of $x$ gives a maximum value of $S$.

**(5)**

(c) Find, to the nearest whole number, the maximum value of $S$.

**(2)**

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**Question 6 continued**

**Question 6 continued**

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**Question 6 continued**

**(Total for Question 6 is 11 marks)**

**7** (a) Solve, in degrees to one decimal place,

$$(3\cos\theta + 5)(5\sin\theta - 3) = 0 \qquad \text{for } 0 \leqslant \theta < 180°$$

**(2)**

(b) Show that the equation

$$8\sin(x - \alpha) = 3\sin(x + \alpha)$$

can be written in the form

$$5\tan x = 11\tan\alpha$$

**(5)**

(c) Hence solve, to one decimal place,

$$8\sin(2y - 30°) = 3\sin(2y + 30°) \qquad \text{for } 0 \leqslant y < 180°$$

**(5)**

**Question 7 continued**

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**Question 7 continued**

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**Question 7 continued**

**(Total for Question 7 is 12 marks)**

**8** (a) Solve $5p^2 - 9p + 4 = 0$

**(2)**

(b) Hence solve $5^{2x+1} - 9(5^x) + 4 = 0$

Give your answers to 3 significant figures where appropriate.

**(4)**

The curve with equation $y = 5^{2x+1} + 5^x$ intersects the curve with equation $y = 2(5^{x+1}) - 4$ at two points.

(c) Find the coordinates of each of these two points.
Give your answers to 3 significant figures where appropriate.

**(4)**

**Question 8 continued**

**Question 8 continued**

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**Question 8 continued**

**(Total for Question 8 is 10 marks)**

**9** (a) Solve the equation $2\log_p 9 + 3\log_3 p = 8$

**(6)**

Given that $\log_2 3 = \log_4 3^k$

(b) find the value of $k$

**(2)**

(c) Show that

$$6x\log_4 x - 3x\log_2 3 - 5\log_4 x + 10\log_2 3 = \log_4\left(\frac{x^{6x-5}}{3^{6x-20}}\right)$$

**(4)**

**Question 9 continued**

**Question 9 continued**

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**Question 9 continued**

**(Total for Question 9 is 12 marks)**

**10** (a) Expand $\left(1 + 2x^2\right)^{-\frac{1}{3}}$ in ascending powers of $x$ up to and including the term in $x^6$, expressing each coefficient as an exact fraction in its lowest terms.

**(3)**

(b) State the range of values of $x$ for which your expansion is valid.

**(1)**

$$\mathrm{f}(x) = \frac{2 + kx^2}{(1 + 2x^2)^{\frac{1}{3}}} \qquad \text{where } k \neq 0$$

(c) Obtain a series expansion for $\mathrm{f}(x)$ in ascending powers of $x$ up to and including the term in $x^6$
Give each coefficient in terms of $k$ where appropriate.

**(3)**

Given that the coefficient of $x^4$ in the series expansion of $\mathrm{f}(x)$ is zero

(d) find the value of $k$.

**(2)**

(e) Hence use algebraic integration to obtain an estimate, to 4 decimal places, of

$$\int_0^{0.5} \mathrm{f}(x)\,\mathrm{d}x$$

**(5)**

**Question 10 continued**

**Question 10 continued**

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**Question 10 continued**

**(Total for Question 10 is 14 marks)**

**11** The curve $C$ has equation $3y = x^2 + 2$

The point $P$ lies on $C$ and has $x$ coordinate 4

The line $k$ is the tangent to $C$ at $P$.

(a) Find an equation for $k$, giving your answer in the form $ay = bx + c$ where $a$, $b$ and $c$ are integers.

**(6)**

The line $l$ is the normal to $C$ at $P$.

(b) Find an equation for $l$, giving your answer in the form $dy = ex + f$ where $d$, $e$ and $f$ are integers.

**(2)**

(c) Find the area of the triangle bounded by the line $k$, the line $l$ and the $x$-axis.

**(3)**

The finite region bounded by $C$, the line $l$, the $x$-axis and the $y$-axis is rotated through 360° about the $x$-axis.

(d) Use algebraic integration to find, to the nearest whole number, the volume of the solid generated.

**(6)**

**Question 11 continued**

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**Question 11 continued**

**(Total for Question 11 is 17 marks)**

**TOTAL FOR PAPER IS 100 MARKS**